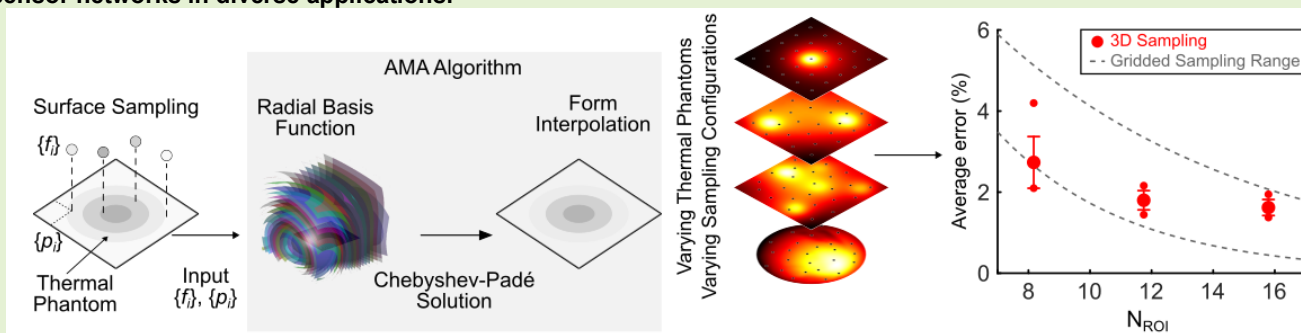


Meshless Interpolation-Based Surface Thermography Characterization for Heat Source Detection

Seo Kyung B. Ahn[†], Jihoon Ahn[†], Tae Hyun Kim, Giuseppe Patanè*, Chiara Daraio*

Abstract— Thermography via distributed sensors is essential in applications ranging from structural health monitoring to medical diagnostics, where localized thermal anomalies reveal subsurface damage or disease. Unlike dense sensor arrays, sparse sensor networks offer advantages such as gas exchange, mechanical conformability, and scalability. However, interpolated temperature maps are sensitive to sensor distribution and interpolation strategy. Many prior methods rely on application-specific data such as heat source location or variograms and lack a generalizable framework for comparing sensor density across scenarios. In this study, we pair a meshless radial basis function-based algorithm, the Adaptive Meshless Approximation (AMA), with a normalized region-of-interest sensor density metric (N_{ROI}) to predict interpolation error without requiring prior knowledge of the ROI . AMA yields temperature fields consistent with diffusive physics, enabling realistic reconstruction. We evaluate performance across gridded and random sampling, multiple phantom geometries, and 3D surfaces. Results show that N_{ROI} strongly correlates with interpolation error across conditions, providing a scalable framework for designing low-density thermal sensor networks in diverse applications.



Index Terms— Chebyshev Approximation, Error Analysis, Function approximation, Interpolation, Numerical Analysis, Metrics, Surface Fitting, Thermal Variables Measurement.

I. Introduction

Accurate surface temperature measurements are critical for different applications such as civil engineering [1], [2], medicine [3], and aerospace engineering [4]. Surface thermography is widely employed for precise measurement, monitoring, and data collection of temperature [5]. Defects, degradation, and internal damages manifest as localized hot spots on the surface of structures and materials [6], [7], [8], [9], [10]. Medical conditions such as ulceration [11], [12], [13], [14], [15], wound infection [16], [17], and deep vein thrombosis [18], [19] are characterized by elevated local skin temperatures. Among thermography methods, direct-contact, sensor-based

thermography, in which physical temperature sensors are distributed across a region of interest (ROI), is widely used [20], [21], [22]. This method offers advantages such as long-term monitoring [23], ability to measure visually obstructed surfaces [24], and integration with soft, flexible systems [13], [25], [26].

The most straightforward approach to sensor-based thermography is to cover the entire region of interest (ROI) with a dense sensor array [26], [27], [28], [29], [30], [31]. While this maximizes spatial accuracy, it introduces several limitations. First, total sensor coverage of the ROI impedes gas and

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moisture exchange, reducing ergonomics for wearable or skin-interfacing applications [20]. Second, sensor arrays are often fabricated on a continuous substrate, which results in folding when applied to dynamic surfaces like joints or various parts of the human anatomy [28]. Third, as the number of sensors increases, so does the risk of crosstalk between channels, along with the complexity of the readout circuitry needed to support high-density measurements [27], [32]. Various breakthroughs such as gas-permeable [20], [33] and flexible substrates [20], [34], and crosstalk-reduction via machine learning [32] have been proposed. However, the fundamental issue remains: sensor count needed to achieve full coverage increases quadratically with *ROI* dimensions, leading to poor scalability.

Interpolation-based sensor networks reconstruct spatial temperature map of the *ROI* from sparsified data [35], [36], [37]. This method offers several advantages over conventional sensor arrays. First, sensors are not constrained to a shared substrate, allowing much of the *ROI* to remain uncovered. This method also enables strategic placement on dynamic surfaces, enhancing suitability for wearables and robotics. Most importantly, reducing the total number of sensors mitigates crosstalk and lowers power consumption [38]. Additionally, modular sensor elements can be replaced individually upon failure, improving the system's robustness overall [39].

The accuracy of interpolated temperature maps produced by sensor networks is greatly affected by both the interpolation algorithm [40], [41], [42] and sensor distribution [36], [43], [44]. For interpolation algorithms, methodologies that produce a continuously differentiable temperature field comparable to heat diffusion behavior outperform simpler methods in interpolation accuracy [42], [45], [46]. For example, a study by Jedermann *et al.* demonstrated that use of Kriging yielded 20% less interpolation error compared to Inverse-Distance Weighting (IDW) in cold-storage applications [47]. Their study showed that supplying Kriging with a known variogram of the region of interest (*ROI*) improved interpolation accuracy in industrial settings. However, variogram-based methods typically assume isotropy and may not account for external variables. Qu *et al.* addressed this limitation by developing a novel deep-learning model for temperature distribution inside of a granary that accounts for spatial heterogeneity introduced by factors such as humidity and pest infestation [48]. Once trained, the model demonstrated generalizability to granaries in similar environmental conditions.

Likewise, the effect of sensor distribution on interpolation accuracy is widely documented [49]. The impact of sensor distribution on interpolation accuracy spans two major dimensions: the spatial placement of sensors relative to thermal features, and the density or number of sensors deployed. In the field of Geographic Information Systems (GIS), foundational studies have shown that areas with steeper thermal gradients require concentration of sensor stations to achieve interpolation fidelity [50], [51]. This principle also applies to smaller distance scales: Woolridge and Bownds demonstrated that implants with smaller diameters, which generate steeper thermal gradients, exhibit greater interpolation errors unless sensor placement is adjusted accordingly [44]. Several efforts have sought to

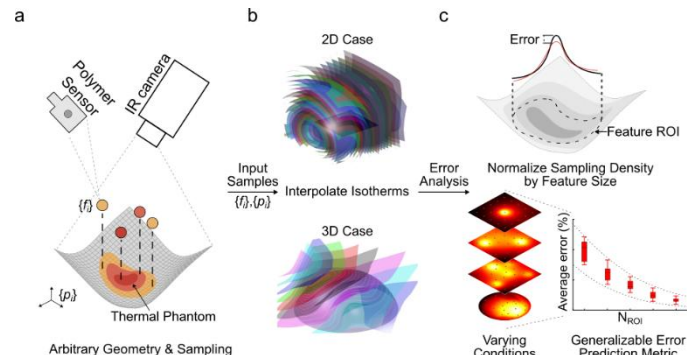


Fig. 1. Workflow visualization for evaluating interpolation performance using the Adaptive Meshless Approximation (AMA). a) Arbitrary thermal phantom shape, sampling configuration, and surface geometry may be used. b) The AMA constructs isotherms on the surface, regardless of dimensions. c) Normalizing sampling density and segmenting the image (*ROI*) allows us to predict interpolation error across varying conditions.

improve sensor placement in such high-gradient regions. Li *et al.* proposed a data-driven approach to sensor placement on a microprocessor die using principal component analysis (PCA), where sensor locations were optimized based on prior temperature maps [43]. This method highlighted how sensor placement affected reconstruction accuracy but required access to representative training data. Other studies have focused on the number of sensors necessary to resolve thermal features. Cochran and Reda proposed a framework to determine the appropriate number of sensors for thermography of CPU dies [52]. By leveraging the spectral sparsity of the temperature map in the Fourier domain, they were able to determine the minimum number of sensors to adequately characterize a band-limited temperature map.

While prior studies in interpolated thermography have made significant advances, they are typically tailored to specific applications and conditions [44], [47], [48], [52]. As a result, the sensor placement strategies proposed in one study may not readily generalize to other scenarios, particularly when the size of thermal anomalies, spatial temperature gradients, or *ROI* geometries differ. This limitation is especially pronounced when comparing sampling densities across studies, since the relative scale of the thermal features being resolved is often not normalized, making direct comparisons ambiguous. Additionally, many interpolation methods employed in prior studies rely on prior knowledge about the *ROI* such as the location of heat sources or a pre-defined variogram. This limits their applicability in scenarios where such information is unavailable or difficult to obtain. As such, there is a critical need for interpolation frameworks that make minimal assumptions about the *ROI* while still producing temperature maps that faithfully model diffusive heat behavior. Furthermore, it is essential to characterize the expected interpolation error as a function of the sampling distribution and the scale of thermal features of interest, thereby providing end users with a predictive framework for evaluating reconstruction reliability under varying sampling and *ROI* conditions.

We present a generalizable framework that is empirically shown to predict thermal interpolation error using a normalized sensor density metric, N_{ROI} , paired with an interpolation algorithm that satisfies the heat diffusion equation and operates

without prior data on the *ROI*. To evaluate interpolation performance, we create thermal phantoms over a surface (Fig. 1a) and apply the *Adaptive Meshless Approximation* (AMA) algorithm [53]. The AMA Algorithm utilizes meshless approximation with radial basis functions to create continuous temperature maps that inherently satisfies the heat diffusion equation without any prior knowledge of the *ROI*'s characteristics (Fig. 1b). Our methodology is initially validated with gridded sampling on a planar surface featuring a single, axisymmetric heat source, and subsequently extended to meshless sampling, complex thermal phantom geometries, and three-dimensional surfaces (Fig. 1c). We demonstrate that our proposed N_{ROI} metric yields a predictable range of interpolation error across various testing conditions, highlighting the robustness of the AMA algorithm and potential generalizability of our evaluation framework.

II. PARAMETER DEFINITIONS

r : radial distance from center of thermal phantom

d : distance between samples

d_{min} : minimum distance between samples for stochastic sampling

A_{sample} : Effective sampling area, defined as $\pi(d/2)^2$. Represents an estimate of the area influenced by a given sample point in the sampling configuration.

T_{Otsu} : Temperature that serves as threshold for defining the *ROI*, where any pixels above T_{Otsu} are included in the *ROI*, determined via Otsu's thresholding.

Ground Truth: Temperature field measured by the IR camera, used as target of comparison for the interpolated maps.

ROI: Region of Interest.

A_{ROI} : Area of the *ROI*.

N_{ROI} : Defined as A_{ROI} / A_{sample} , normalized sampling density metric.

ΔT : Absolute range of temperature present on the temperature map

$\% \epsilon$: Relative interpolation error, expressed as percentage of ΔT .

Center Sampled: Datasets where a sample taken from the center of the thermal phantom, and therefore the approximate temperature maxima, is guaranteed.

Center Unsamed: Datasets where we ensure we do not take a sample from the center of the thermal phantom, and therefore the approximate temperature maxima, is not guaranteed.

Peak Region: Area of the *ROI* marked by minimal temperature change close to the origin of the thermal anomaly. Bounded by the area of the *ROI* where the second derivative of temperature approaches zero.

HG Region: Area of the *ROI* marked by high thermal gradients, defined as the area of the *ROI* not included within the *Peak Region*.

ADW: Angle-Distance Weighting Method

S_{max} : Temperature of the highest sampled point, expressed as percentage of the maximum temperature from ground truth.

T^* : Isotherms that disproportionately affect the interpolation accuracy found in the *HG Region*.

T_s : Sampled temperature

T_{diff} : Absolute difference between T^* and T_s

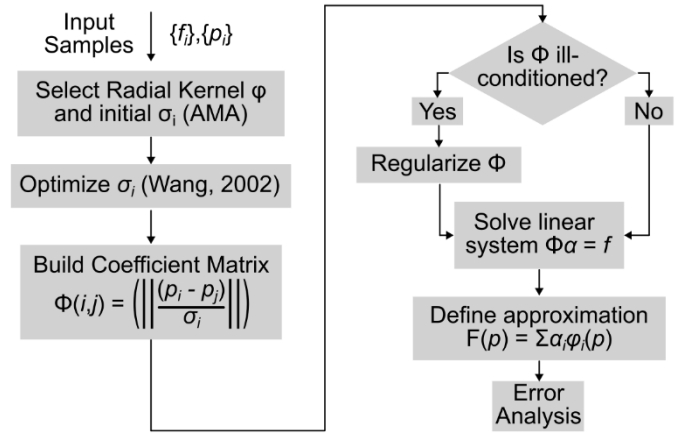


Fig. 2. Workflow of the Adaptive Meshless Approximation (AMA).

III. AMA METHOD

The *Adaptive Meshless Approximation* (AMA) framework is a local, physics-constrained meshless solver for reconstructing temperature fields from sparse measurements. Unlike classical global RBF interpolation methods, which construct a single dense approximation primarily to minimize geometric interpolation error, AMA formulates the reconstruction as a local collocation solution of the heat diffusion equation, using radial basis functions as trial functions. The temperature field is approximated as

$$T(x) = \sum_{j=1}^N \alpha_j \phi \left(\frac{\|x - x_j\|}{s_j} \right) + p(x), \quad (1)$$

where ϕ is a smooth radial kernel, x_j are adaptively selected centers, s_j are shape parameters, and $p(x)$ is a low-degree polynomial term ensuring exact reproduction of constant and linear fields. In contrast to global RBF solvers, AMA does not solve a single dense linear system over all samples. Instead, it constructs compact local stencils, selecting only the nearest and most informative neighbouring centres based on the variability of the underlying temperature field. The stencil size and composition are adapted to preserve the global structure of the signal before introducing local refinements. This has the advantage of allowing globally supported kernels without requiring clustering [54], [55], kernel methods [56], sampling [57], [58], or sparsification [59]. Furthermore, this strategy enhances numerical conditioning, prevents over-smoothing in under-sampled regions, and enables real-time computation on both CPU and GPU architectures. Furthermore, we construct the approximation so that the heat diffusion operator acts directly on the basis functions. Denoting

$$\mathcal{L}[T] = \nabla \cdot (k(x) \nabla T(x)) \quad (2)$$

as the steady heat operator with thermal conductivity $k(x)$, and assuming that the generating kernel ϕ is sufficiently smooth so that the differential operators act termwise on it, the AMA enforces

$$\mathcal{L}[T](y_i) = q(y_i) \quad (3)$$

on the collocation points $\{y_i\}$, while Dirichlet or Neumann boundary conditions are enforced at boundary collocation points. By solving small local linear systems to obtain the resulting coefficients, we obtain an approximation that satisfies the diffusion equation exactly at the collocation points and

approximately elsewhere. Furthermore, by including the polynomial term and enforcing the discrete moment conditions, the AMA method ensures that the PDE residual is orthogonal to constants, implying

$$\int_{\Omega} (\mathcal{L}[T] - q) dx = 0. \quad (4)$$

By the divergence theorem, this condition guarantees that the net boundary heat flux equals the total internal heat generation. Therefore, the resulting temperature field is physically meaningful, with gradients and fluxes derived from the interpolated field being consistent with diffusive heat transport.

Finally, the AMA method does not require topology structures, neighbor lists, or element connectivity, which reduces overhead and results in a practical balance between physical accuracy and computational efficiency. Strategies such as using compactly supported kernels and GPU acceleration via parallelizing kernel evaluations may be used to significantly reduce the runtime [60], [61]. More detailed explanations can be found in the Supplementary Materials.

IV. EXPERIMENT SETUP

A. Workflow

We evaluate the performance of the AMA algorithm by comparing the interpolated temperature maps against infrared (IR) thermal imagery, which serves as the ground truth. To generate the ground-truth temperature field, the sample surface is coated with black dye to achieve near-unity emissivity (Fig. 3a); the emissivity change and the corresponding increase in measured IR temperature accuracy is verified through corresponding changes in measured temperature (Fig. 3b). Thin plate-like surfaces are selected because reduced thickness produces steeper surface thermal gradients (Fig. S1), which are associated with increased interpolation error. Accordingly, thin samples provide a conservative test condition for evaluating interpolation performance.

Thermal anomalies are generated by establishing conductive heat transfer from a hot plate to the sample surface using an aluminum conduction rod (Fig. 3c). Non-axisymmetric temperature fields are produced by placing multiple aluminum rods at stochastic locations using the Poisson Disc Distribution [62], [63]. For three-dimensional surfaces, localized heating is applied using a flexible heating pad.

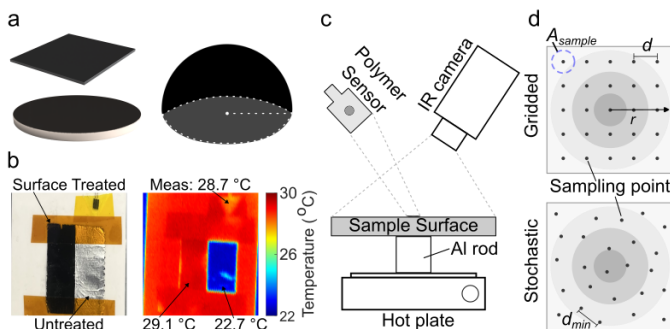


Fig. 3. Experiment Setup. (a) Sample surfaces used: Aluminum plate (1.5mm thick), Ecoflex disc (10mm thick), steel hemisphere (2 mm thick). (b) Emissivity control via surface treatment shows quantifiable IR measurement improvement. (c) Schematic of the thermal phantom and sampling methods. (d) Examples of gridded and stochastic sampling.

Discrete sampling coordinates are defined by the sampling distance d and the sampling configuration (gridded or stochastic). Temperature measurements are obtained either by extracting pixel values from the IR image or by affixing a temperature sensor at the corresponding locations (Fig. 3d). Interpolation is performed using the measured temperatures and sample coordinates, and the mean interpolation error is computed over the region of interest (*ROI*), which is segmented using Otsu's thresholding method [64]. Detailed methods may be found in Supplementary Materials.

V. RESULTS AND DISCUSSION

A. Effect of Sampling Configuration on Error

We first examine the effect of thermal anomaly magnitude on the interpolation error by inducing a thermal phantom on an Aluminum sheet at prescribed temperature setpoints (30 °C–60 °C in 5 °C increments), producing a ground-truth temperature map with a well-defined region of interest (*ROI*) (Fig. 4a). AMA interpolation is performed using temperature samples extracted from IR imagery on a center-sampled grid with varying sampling distances (Table I). After normalizing the ground truth by ΔT , the results indicate that sampling density is the dominant factor governing interpolation quality, largely independent of anomaly magnitude (Fig. 4b).

The results indicate (1) defining *ROI* boundaries using Otsu's thresholding and (2) normalizing the heat source by ΔT yield comparable mean relative interpolation error (% ϵ) within the *ROI* across different temperature magnitudes. Because the Otsu thresholding boundary remains invariant across heat-source temperature intervals (Fig. 4a), these results support the hypothesis that the proposed normalization enables quantitative comparison of % ϵ across cases with varying surface thermal gradients, provided that the size of the thermal anomaly remains constant. This agreement aligns with simplified analytical estimates suggesting that, under diffusion-dominated conditions where nonlinear terms such as convection are negligible, the thresholding boundary is expected to be temperature invariant (Supplementary Materials).

Furthermore, the mean *ROI* % ϵ exhibits diminishing returns with respect to the normalized sampling density metric N_{ROI} . Specifically, a 3.48 $\times$ increase in N_{ROI} from 3.21 to 11.18 reduces the mean % ϵ by 6.5%, whereas a comparable 3.52 $\times$ increase from $N_{ROI} = 11.18$ to 39.3 yields only a 0.76% reduction. This trend is consistent with prior studies reporting a nonlinear relationship between interpolation accuracy and sampling density [64]. Going forward, we normalize the absolute temperature of the heat source and examine other factors that affect interpolation.

To assess whether the trend between % ϵ and sampling distance d extends beyond a constrained scenario where interpolation is based on samples on a gridded configuration

Table I

Sampling distance d and corresponding N_{ROI} values

$d(\text{mm})$	10	15	18.75	25	30	35	40
N_{ROI}	39.3	17.5	11.2	6.3	4.4	3.2	2.5

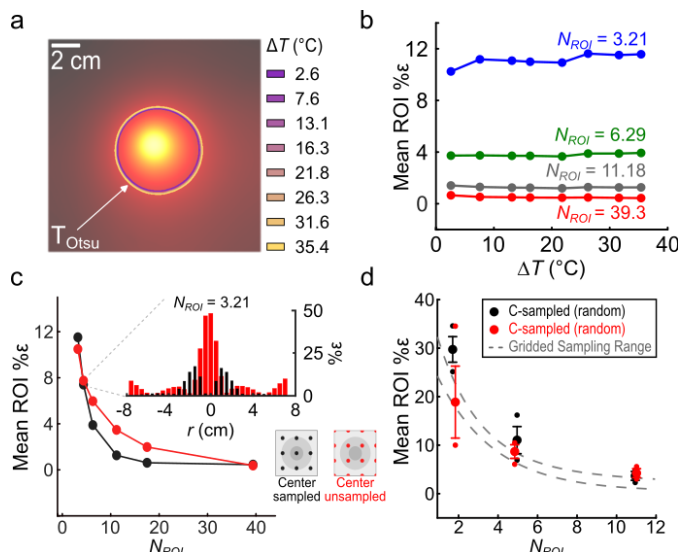


Fig. 4. Experiments on 1.5mm Al plate. (a) ROI bounds defined by T_{Otsu} at varying ΔT values remains mostly invariant. (b) Relationship of Mean ROI % ϵ vs ΔT , with changing normalized sampling density N_{ROI} . (c) Mean ROI % ϵ vs N_{ROI} on Center Sampled and Center Unsampled datasets. Inset shows N_{ROI} values where the Center Unsampled data outperforms Center Sampled. (d) Mean ROI % ϵ vs N_{ROI} on interpolations based on stochastic datasets, with small dots showing the range and whiskers showing the standard deviation.

that directly captures the maximum temperature, we examine the effect of sampling configuration on interpolation accuracy.

Offsetting the sampling coordinates to maximize the distance from the temperature maximum (*Center Unsampled*) yields a similar relationship between N_{ROI} and % ϵ as observed for the *Center Sampled* configuration (Fig. 4c). Notably, the N_{ROI} –% ϵ curves for the two sampling configurations intersect, indicating that the *Center Unsampled* configuration can outperform the *Center Sampled* configuration over specific N_{ROI} ranges (Fig. 4c). This observation suggests that interpolation accuracy is not solely determined by directly sampling the temperature maximum.

To further examine this behavior, we bin % ϵ as a function of radial distance r to generate error histograms as seen in the inset of Fig. 4c. These distributions reveal spatial domains in which the *Center Unsampled* configuration exhibits lower % ϵ than the *Center Sampled* configuration, indicating that additional regions on the surface play a significant role in governing interpolation accuracy.

We introduce stochasticity into the sampling configuration to (1) assess its effect on % ϵ and (2) identify influential spatial regions. Sample points are generated using a Poisson disk distribution with a minimum separation distance d_{min} . Interpolation using temperature values extracted from IR imagery exhibits trends nearly identical to those observed under gridded sampling, irrespective of whether the temperature maximum is sampled (Fig. 4d).

The data indicates the existence of a critical sampling density threshold beyond which interpolation over a single thermal anomaly rapidly deteriorates, independent of sampling configuration. This transition occurs at approximately $N_{ROI} \approx 10$. Below this threshold, random variation in sample placement disproportionately affects interpolation accuracy, as evidenced by a sharp increase in the standard deviation in % ϵ for $N_{ROI} < 4$.

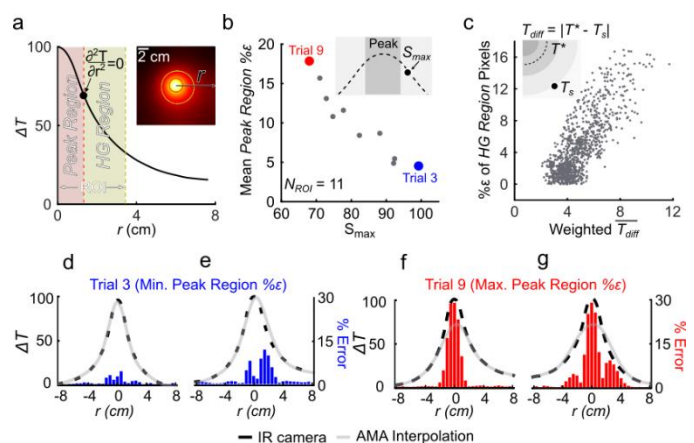


Fig. 5. Investigation on effect of sampling strategy on % ϵ . $N_{ROI} = 11$ for all data. (a) The ROI subdivided into the Peak and HG Region. (b) S_{max} vs Mean Peak Region % ϵ . (c) % ϵ of randomly selected pixels in the HG Region vs mean T_{diff} weighted by the ADW method. Inset shows relation between T^* and T_s . Temperature profiles and % ϵ vs r plots of Center Unsampled interpolation trials for (d,e) Trial 3 (lowest Peak Region % ϵ) and (f,g) Trial 9 (highest Peak Region % ϵ). (d,f) correspond to max HG Region % ϵ temperature profile and (e,g) correspond to min HG Region % ϵ temperature profile within given trial, respectively.

These findings indicate that N_{ROI} can provide practical guidance for sampling density selection to improve interpolation fidelity in thermographic reconstruction of comparable thermal anomalies. Compared to absolute sampling density, N_{ROI} accounts for interpolation domain size and mitigates artifacts such as artificially inflated density estimates arising from smaller interpolation regions (Fig. S2). Notably, individual trials at $N_{ROI} \approx 2$ exhibit interpolation errors comparable to those at $N_{ROI} \approx 4.5$, indicating that in those individual cases, specific sample placement can partially mitigate increase in % ϵ caused by sampling sparsity. The following section examines the role of sample placement on interpolation fidelity.

B. Effect of Sampling Strategy on Error

To investigate the elevated variance in % ϵ observed under stochastic sampling, the ROI is subdivided into the Peak and HG Regions (Fig. 5a), as defined in Section II (Fig. S3). Random sampling is performed over 10 trials, and the mean % ϵ is evaluated within each subregion.

The mean interpolation error ($N_{ROI} = 11$) in the Peak Region is plotted against the Temperature of the highest sampled point, S_{max} , for each trial (Fig. 5b), revealing a strong linear correlation ($R^2 = 0.92$). In contrast, the Peak Region mean ROI % ϵ plotted against the distance between the anomaly center and the sample corresponding to S_{max} exhibits a weaker, nonlinear relationship ($R^2 = 0.825$; Fig. S4). These results indicate that sampling metrics normalized by the relative size of the thermal feature, rather than absolute distance-based metrics, provide more generalizable guidance for interpolation-based sampling strategies.

Qualitative analysis shows that certain regions within the HG Region disproportionately affect % ϵ (Fig. S5). From the HG Region, 100 pixels are randomly selected per trial ($\Delta T = 24.5$ °C, $N_{ROI} = 11$, Center Sampled), excluding pixels within a radius of $r = 20$ px of sampled points due to their negligible interpolation error. The mean % ϵ over the HG Region is then

plotted against the ADW-weighted T_{diff} evaluated at each selected pixel (Fig. 5c). The ADW assigns a weight to each sample point according to spatial proximity and directional alignment relative to the selected pixel [65]; the full formulation and justification are provided in the Supplementary Materials.

By iterating over values of T^* , a peak correlation ($R^2 = 0.614$) is observed between mean T_{diff} and interpolation error at $T^* = 0.475 T_{max}$, where T_{max} denotes the maximum temperature in the ground-truth map. The comparatively weaker correlation is attributed to the distance-weighting component of ADW, which disproportionately amplifies the influence of S_{max} (Fig. S6a,b). Consistent with this interpretation, applying the same analysis to the *Center Unsampld* dataset yields a lower peak correlation ($R^2 = 0.47$) and a shifted optimal value of $T^* = 0.4$, reflecting increased variability in the placement of S_{max} (Fig. S6c,d).

These results highlight that interpolation accuracy depends not only on sampling density, but also on the radial distribution of sensor locations, highlighting the role of sampling strategy. Moreover, the sensitivity of interpolation accuracy to sampling near T^* provides a mechanistic explanation for the increased variance observed at $N_{ROI} \approx 2$.

Finally, two temperature profiles are selected from each of two trials exhibiting the lowest (Trial 3) and highest (Trial 9) interpolation error in the *Peak Region*. For each trial, one profile passes through a pixel with low error and another through a pixel with high error in the *HG Region*. In the *Peak Region*, the maximum interpolation error is limited to 4% for Trial 3 (Fig. 5d) but exceeds 28% for Trial 9 (Fig. 5f). In the *HG Region*, the maximum percent error ranges from <1% (Figs. 5d,f) to as high as 8% (Figs. 5e,g), depending on local sampling conditions.

These error ranges arise despite all four profiles being generated from interpolations using the same d_{min} (and therefore comparable N_{ROI}). Moreover, both Trials 3 and 9 exhibit spatially varying interpolation accuracy within the *HG Region*. Collectively, these observations indicate a partial independence between percent error in the *Peak Region* and that in the *HG Region* beyond localized edge effects. Consequently, directly sampling the absolute maximum temperature of the heat source is not a strict requirement for achieving high interpolation accuracy across the *ROI* using the AMA algorithm.

C. Effect of Geometry on Error

The preceding sections examined factors influencing %e beyond sampling density for a single, axisymmetric thermal anomaly. Here, we investigate the impact of thermal anomaly geometry and measurement surface geometry on %e and assess whether the findings from the previous sections extend to cases involving non-axisymmetric thermal anomalies and nonplanar sampling surfaces.

First, irregular and asymmetric thermal anomalies are generated using multiple heat-conduction rods. In the two-source configuration, the *HG Regions* of the individual thermal phantoms are arranged to overlap (Fig. 6b), whereas in the five-source configuration the *Peak Regions* overlap (Fig. 6c). Poisson disk sampling is then used to generate 10 independent trials at each N_{ROI} value ($N_{ROI} = 11, 4.9, 1.8$) for each heatmap. Identical d_{min} values are applied across all maps to ensure consistent sampling density. To avoid introducing an arbitrary *ROI* size, N_{ROI} is computed using the *ROI* area from the single-

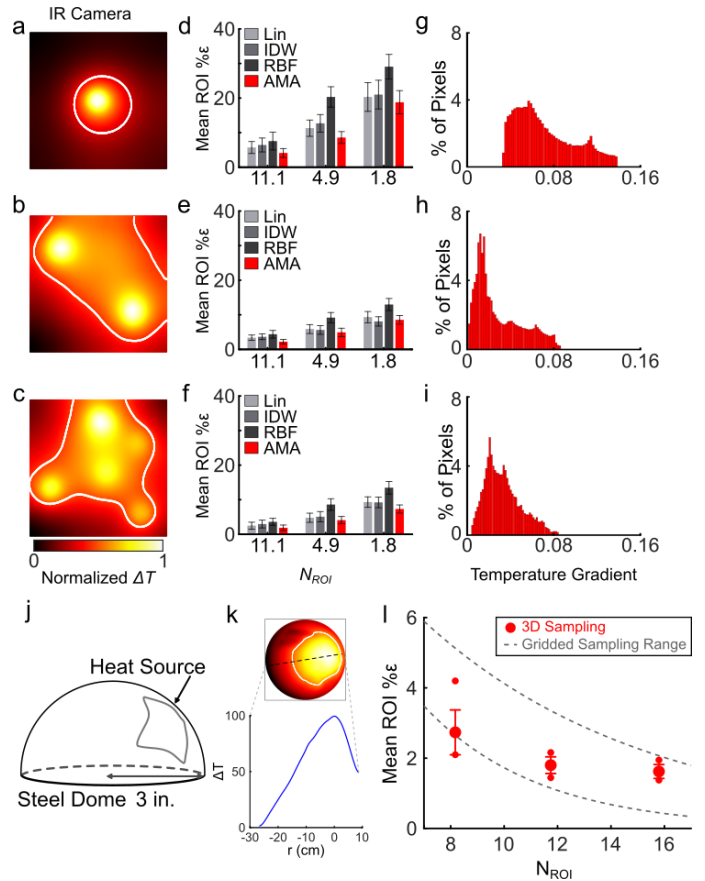


Fig. 6. Investigation on effect of thermal anomaly and surface geometry on %e. (a-c) Heatmap of 1 source (a), 2 source (b), and 5 source (c) maps with corresponding ROIs. (d-f) Mean ROI %e of linear, inverse distance weighted, radial-basis function, and AMA interpolation Whiskers show the standard deviation over 10 interpolations. (g-i) Distribution of the thermal gradient of pixels within the ROI. (j) Thermal anomaly generation on hemispherical surface. (k) Top view of heat map and asymmetrical temperature profile generated (l) Mean ROI %e vs N_{ROI} relation on the 3D surface Whiskers show the standard deviation over 10 trials.

source map, since thermal phantoms in the multi-source cases are placed stochastically.

Mean interpolation error computed over the *ROI* defined by Otsu's thresholding decreases with increasing numbers of thermal anomalies (6d-f). Across all tested N_{ROI} values, the single-source map exhibits mean interpolation errors more than twice those of the two-source and five-source maps. Furthermore, the AMA algorithm outperforms standard interpolation approaches, including bilinear interpolation [72], inverse distance weighting (IDW) [73], and radial basis function (RBF) interpolation [66].

The observed inverse relationship between interpolation accuracy and heatmap complexity can be attributed to the thermal gradient, which is a primary determinant of interpolation accuracy [36]. A histogram of the thermal gradient within the *ROI* (Fig 6g-i) shows that overlap between the *Peak* and *HG Regions* reduces the overall thermal gradient across the *ROI* (Fig. S7). This observation is significant because it suggests that, when a complex heat source can be approximated as a superposition of multiple singular heat sources, the resulting interpolation error is bounded above by the

interpolation error associated with the corresponding single-source case.

The standard deviation of the mean interpolation error exhibits reduced sensitivity to changes in N_{ROI} for complex heatmaps compared to singular thermal phantoms (Fig 6d-f). These results suggest that heatmaps containing fewer high-gradient regions yield lower variance in interpolation error when samples are placed stochastically. This trend is consistent with the findings of the previous section, where sample placement within the *HG Region* was identified as the primary contributor to inter-trial variance in mean *ROI* interpolation error at low N_{ROI} values.

We next examine the effect of nonplanar geometry on interpolation error. A noncircular heat source is generated by affixing a flexible heating pad to a steel hemisphere at a random location. Upon application of a DC voltage, the surface reaches a maximum temperature of 65 °C (Fig. 6j). Sampling points at each N_{ROI} value are generated in a manner consistent with previous experiments, with the distinction that separation distances are defined using geodesic distance along the hemispherical surface. Additional details regarding the experimental setup are provided in the Supplementary Materials.

The resulting interpolation (Fig. 6k) follows the relationship between mean interpolation error and N_{ROI} established in previous experiments (Fig. 6l). This observation supports the generalizability of N_{ROI} as an error prediction metric for thermography and demonstrates its applicability to thermal anomalies on nonplanar surfaces.

Furthermore, this experiment partially addresses the influence of convective effects on the proposed methodology. In our previous work [53], nonlinear terms associated with convection were identified as a potential source of instability. By demonstrating that interpolation performed on a non-planar sample surface with increased surface area exhibits a similar interpolation $\% \epsilon$ - N_{ROI} relationship to that observed for planar surfaces, the results suggest that geometry-induced variations in convection have a limited impact on interpolation error prediction under the tested conditions.

D. Effect of Sensor Noise on Error

Finally, we apply the proposed analysis methodology to data acquired from physical sensors. In prior studies, the methodology was evaluated exclusively using sampled infrared (IR) pixels as surrogate discrete sensor elements, which do not capture practical sources of error such as measurement noise, hysteresis, and drift.

A sensor element was fabricated by drop-casting a temperature-sensitive polymer (P3A) onto a fringing Au-Ti electrode [67] (Fig. 7a). An axisymmetric thermal phantom was generated on a 10-mm-thick Ecoflex 00-20 silicone rubber disk, and the polymer sensor was used to sample surface temperature at fixed spatial coordinates ($d_{min} = 1$ cm, $N_{ROI} = 39$) as shown in the inset of Fig. 7b. Sensor readout was obtained using an impedance analyzer (MFIA, Zurich Instruments), with temperature inferred via a calibrated temperature-impedance relationship To characterize hysteresis-induced drift, the sensor was subjected to a controlled thermal cycling at fixed humidity, from which the temperature-impedance relationship and associated drift behavior were extracted (Fig. 7b).

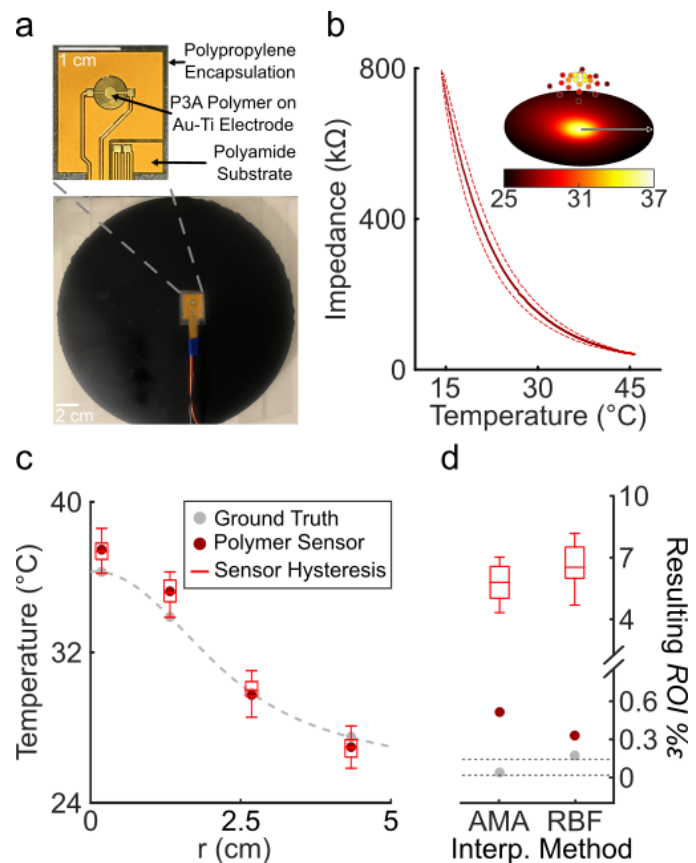


Fig. 7. Investigation of effect of sensor inaccuracy on interpolation fidelity. (a) P3A polymer drop cast on Au-Ti electrode serving as sensing element. (b) Calibration curve of sensor in Impedance (kΩ) vs Temperature (°C), hysteresis range shown in dotted line. Inset shows sample placement location and temperature profile for plot in 7c. (c) Observed and measured temperature along temperature profile. Drift due to hysteresis shown as box plot, whiskers indicating standard deviation. (d) Mean *ROI* $\% \epsilon$ relation for interpolations based on IR pixel samples and sensor-measured samples, before and after hysteresis-induced drift. Error range corresponding to similar N_{ROI} from Figure 3 shown in black.

A comparison between infrared (IR) temperature measurements and temperatures acquired using the polymer sensor, before and after hysteresis-induced drift (20 trials, in red), reveals increased measurement error in regions of elevated surface temperature (Fig. 7c). This behavior can be attributed to several factors. First, the nonlinear impedance-temperature relationship results in larger temperature uncertainty at higher temperatures, where identical impedance variations correspond to greater temperature variance than at lower temperatures. Second, the finite size of the sensing element relative to the local thermal gradient contributes to spatial averaging error. The sensor has an effective diameter of approximately 0.5 cm, which is sufficient for temperature variations on the order of 1 °C to occur across its footprint in the *HG Region*.

AMA and radial basis function (RBF) interpolation were subsequently applied to discrete sampling points extracted from both the IR images and the polymer sensor measurements (Fig. 7d). For interpolation based on IR-derived samples, the AMA method ($\% \epsilon$: 0.02%) outperforms standard RBF interpolation with a Gaussian kernel ($\% \epsilon$: 0.15%). When sensor noise is introduced, the AMA method exhibits a larger relative increase in $\% \epsilon$ compared to RBF interpolation using the same data. Finally, the introduction of several degrees of hysteresis-

induced drift leads to severe degradation of interpolation accuracy for both methods.

The observation that the AMA's % ϵ falls within the range predicted by gridded sampling (black) on a thin aluminum plate (previously shown in Fig. 4d) supports the hypothesis that the proposed analysis framework can be extended to surfaces with varying thermal conductivities and heat source depths. The addition of measurement noise produces only minor deviations from the error range observed in prior experiments. In contrast, the introduction of sensor drift results in substantial degradation of interpolation performance. We attribute this sensitivity to the AMA forming its interpolation using only sample coordinates and temperature values. Although this avoids the need for prior material properties or variogram models, it also allows measurement errors to propagate directly into the reconstructed field. Overall, we expect our predictive framework to remain valid if sampling measurement error is kept to less than 5% of the ΔT value.

VI. CONCLUSIONS

This study presents an error prediction framework for interpolation-based thermography and systematically evaluates a physics-constrained meshless algorithm (AMA) across diverse sampling strategies and surface conditions. Within the scope of the experimental conditions examined, the results demonstrate that the proposed error prediction approach and the normalized sampling density metric N_{ROI} provide consistent trends for estimating interpolation error across both structured and stochastic sampling schemes. Minimum N_{ROI} thresholds associated with reliable interpolation and approximate isotherms with oversized effect on interpolation accuracy (T^*) are identified for the tested configurations.

The proposed framework demonstrates predictive capability for static and quasi-static heat sources. However, the scope of this study is limited to these conditions and does not assess the applicability of the method to rapidly time-varying temperature fields encountered in dynamic thermal environments.

By providing quantitative guidance on sampling density and sample placement within the tested regimes, this work informs the design of interpolation-based thermography systems seeking to resolve thermal features across multiple spatial scales while reducing the number of required sensing elements. Future work could focus on extending the framework to a broader range of materials and convective environments, as well as incorporating broader datasets from physical sensors to improve robustness under realistic measurement conditions.

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